18.01A Recitation — Wednesday, Oct. 3, 2018

Left from last time:

$$\int \frac{3x^2 - x + 4}{x^3 + 2x^2 + 2x} dx$$

Quick Review on improper integral:

- Why are they "improper"?

  Possibility 1: limits of the integral contain ∞. e.g., $\int_0^\infty 1/x dx$

  Possibility 2: the integrand goes to $\pm\infty$ inside the domain of the integral. e.g., $\int_0^1 1/x dx$.

  The two possibilities can occur in one single integral. e.g., $\int_0^\infty 1/x dx$. When this happens, you need to separate the "problems", e.g., $\int_0^\infty 1/x dx = \int_0^1 1/x dx + \int_1^\infty 1/x dx$ and study the two improper integrals separately. The original improper integral converges ONLY when both of the two separated improper integrals are convergent.

- Two major methods to test for convergence: direct comparison and asymptotic convergence. To use either of them, you need to find a *suitable* function that is comparable to the original integrand around the "problematic" point (the point that makes the integral improper).

- Important improper integrals (the ones that are usually compared against):

- Direct comparison and asymptotic convergence only work when the integrand is always positive/negative, i.e., it cannot change sign.

Practice problems:
Determine whether the following improper integral converges.

1. $$\int_1^\infty e^{-\sqrt{x}} dx$$

2.

$$\int_0^{\infty} \frac{\sqrt{x^3+3x+2}}{\sqrt[3]{x^8+1}}dx$$

3.

$$\int_0^{1} \frac{dx}{x^2+\sqrt{x}}$$