18.01A Recitation — Wednesday, Oct. 10, 2018

Quick Review on infinite series

- the "Sigma" $\sum$ notation, what does convergence mean
- If $\lim_{n\to\infty} a_n \neq 0$, then $\sum a_n$ is divergent. (Think of this as a way to sift out those series that are "terribly" divergent.) — works for any series
- Tests for convergence/divergence: **integral test** (when the general term is decreasing), **direct comparison, asymptotic comparison** (these two work similarly to those for improper integrals), **ratio test** (this test reveals nothing if the limit of the ratio is precisely 1) — works for only series whose general terms do not change sign.
- For series with infinitely many positive terms and infinitely many negative terms: alternating series test.
- Absolute convergence v.s. conditional convergence

Practice problems: Determine whether the following series converge or diverge.

1.
$$\sum \left(1 - \cos\left(\frac{1}{n}\right)\right)$$

2.
$$\sum \frac{n^2 + 3n - 7}{n^3 - 2n + 5}$$

3.

$$\sum \frac{2n+2}{3^n (n!)^2}$$

4. Absolute convergence or conditional convergence:

$$\sum (-1)^{n+1} \frac{1}{n}$$

5.

$$\sum \frac{1}{(1+1/n)^n}$$