18.01A Recitation — Monday, Oct. 15, 2018

Quick Review on Power Series

- It looks like
$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \dots$$
- Finding the interval where it converges: *ratio test* **implies**
$$\text{Radius of Convergence } R = \lim_{n\to\infty} \left|\frac{a_n}{a_{n+1}}\right|$$
- For $x = \pm R$, plug $x = \pm R$ into power series and test them using the convergence test you learnt before. (You will have one series with all positive terms, the other with alternating terms.)
- Taylor's formula: *direct computation*, *substitution*, *differentiate known Taylor's formula*, *integrating known Taylor's formula*
- Use Taylor's formula to solve ODE. (This sounds fancy, but is *extremely* easy/simple-minded.)

Practice problems:

1. Consider the power series:
$$\sum \frac{x^n}{\ln(1+n)}.$$
   (a) Find its radius of convergence $R$.

   (b) Find the interval on which the power series is convergent.

2. Find the Taylor's formula for

$$F(x) = \int_0^{x^2} \cos t dt$$

near $x = 0$ by using the Taylor's formula

$$\cos t = \sum_{n=0}^{\infty} (-1)^n \frac{t^{2n}}{(2n)!}.$$

3. Suppose we have a function $f(x)$ which satisfies $f(0) = 0$ and satisfies the differential equation $f'(x) = x - f(x)$. Write down a formula that relates the $n$-th Taylor coefficient $a_n$ of $f(x)$ in terms of the coefficients $a_m$ with $m < n$. Use this to calculate the first 5 terms of the Taylor series expansion for $f(x)$ near $x = 0$.