18.02A Recitation — Wednesday, Nov. 28, 2018

Quick Review

- Chain rule.
  - Suppose $w = f(x, y)$ and $x = x(t)$, $y = y(t)$. Then
$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt}.$$
  - Supose $w = f(x, y)$ and $x = x(u, v)$, $y = y(u, v)$. Then
$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u},$$
$$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v}.$$
- Implicit differentiation. The key is to keep track of which variable depends on which.

Practice problems:

1. If $u = x^2 - 2y^2 + z^3$ and $x = \sin t$, $y = e^t$, $z = 3t$, find $\frac{du}{dt}$.

2. If $w = x^3 y^5$, and $x = u + v$, $y = u - v$, find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$.

3. If z is implicitly defined as a function of x and y by $x^2 + y^2 - z^2 = 3$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

4. If z is implicitly defined as a function of x and y by $x \sin z - z^2 y = 1$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

5. If z is implicitly defined as a function of x and y by $x^2 + y^2 + z^2 = 1$, show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z - \frac{1}{z}$.