18.02A Recitation — Monday, Dec. 3, 2018

Quick Review

- Double integral

$$\iint_R f(x,y)dA$$

  - Geometrically, if $f(x,y) \geq 0$, this is the volume of the solid under the graph $z = f(x,y)$ over the region R.
  - Algebraically, this is defined by Riemann sum (similar to that in 18.01A).

- Compute a double integral by converting it to an iterated integral.

  1. Draw the region R first! (Very important! Do it!)
  2. Slice the region R either vertically or horizontally.
  3. If slicing vertically: a) figure out the smallest and the largest x in R—your answer should be $a < x < b$; b) now focus on a specific (but generic) x and ask yourself what the range of y—your answer should be $g(x) < y < h(x)$.

     If slicing horizontally: a) figure out the smallest and the largest y in R—your answer should be $a < y < b$; b) now focus on a specific (but generic) y and ask yourself what the range of x—your answer should be $g(y) < x < h(y)$.
  4. Do the iterated integral—inner integral first, then outer integral. Remember, at the end of the day, you get a number!

- Applications:

  - Area of R:

$$\iint_R 1dA$$

  - Volume between $z = f(x,y)$ and $z = g(x,y)$ (with $f(x,y) \geq g(x,y)$) over R:

$$\iint_R \left(f(x,y) - g(x,y)\right) dA$$

  - Average value of f over R:

$$\frac{1}{\text{Area}(R)} \iint_R f(x,y)dA$$

  - Total mass of a metal plate with density distribution $\rho(x,y)$:

$$\iint_R \rho(x,y)dA$$

– Center of mass of a metal plate with density distribution $\rho(x, y)$:

$$x_0 = \frac{1}{\text{mass}} \iint_R x\rho(x, y) dA$$

$$y_0 = \frac{1}{\text{mass}} \iint_R y\rho(x, y) dA$$

Practice problems:

1. Evaluate the following double integral

$$\iint_R y dA,$$

where R is the triangle with vertices at $(\pm 1, 0)$, $(0, 1)$.

2. Evaluate the double integral

$$\iint 2x + 4y dA,$$

where R is the region bounded by $y = \sqrt{x}$ and $y = x^2$.

3. Evaluate each of the following iterated integrals by changing the orders of integration. (Start by sketching R.)

   a)

$$\int_0^{\frac{1}{4}} \int_{\sqrt{t}}^{\frac{1}{2}} \frac{e^u}{u} du dt$$

   b)

$$\int_0^1 \int_{x^{1/3}}^1 \frac{1}{1+u^4} du dx$$

4. Find the volumes above the xy-plane bounded by the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 2$.