18.02A Recitation — Monday, Oct. 29, 2018

Quick Review

- Line: a point on the line $P_0 = (x_0, y_0, z_0)$ and a direction $\mathbf{a} = \langle a_1, a_2, a_3\rangle$
  1. Parametric equation: $\langle x, y, z\rangle = \langle x_0, y_0, z_0\rangle + t\langle a_1, a_2, a_3\rangle$.
  2. Symmetric equation: $\frac{x-x_0}{a_1} = \frac{y-y_0}{a_2} = \frac{z-z_0}{a_3}$.
- Plane: a point on the plane $P_0 = (x_0, y_0, z_0)$ and a normal vector $\mathbf{b} = \langle b_1, b_2, b_3\rangle$

$$\langle x - x_0, y - y_0, z - z_0\rangle \cdot \mathbf{b} = 0,$$

  or

$$b_1(x - x_0) + b_2(y - y_0) + b_3(z - z_0) = 0.$$

- Intersections of two planes. Three situations:
  1. *Parallel*, but non-intersecting: *the two normal vectors are proportional*, but take an arbitrary point in one plane and that point does not belong to the other plane.
  2. the two planes are identical: the two normal vectors are proportional, and if you take an arbitrary point in one plane then it belongs to the other plane as well.
  3. non-parallel and intersecting. The intersection is a line. This happens when the two normal vectors are not proportional.
     - To find the intersecting line, cross product of the two normal vectors (of the planes) gives you the direction of the line; to get a point on the line, solve the equations of the two planes (you have 2 equations and 3 variables, so you get to choose one variable for free!!).
     - Angles between two planes: angles between the two normal vectors.
- Intersection of a line and a plane. Three situations:
  1. The line is parallel to the plane, but not on the plane: normal of the plane is perpendicular to the direction of the line, and an arbitrarily chosen point on the line is not on the plane.
  2. The line is on the plane: the parametric line equation satisfies the plane equation.
  3. The line punctures through the plane: normal of the plane is not perpendicular to the direction of the line. To find the point of intersection, use parametric line equation and the plane equation.

Practice problems:

1. Find the equation of the plane P passing through the points $A = (1, 0, 0)$, $B = (0, 1, 0)$, and $C = (0, 0, 1)$.

2. Find the equation of the line passing through $(0, -1, -1)$ and $(2, 3, 3)$. Does it intersect with the plane P in Question #1? If there is any intersection point, find that.

3. Consider two planes $P_1 : x - y + z = 1$ and $P_2 : x + y + z = 1$. Are they parallel to each other? Are they the same plane? is their intersection a line? If it is a line, find the quation to that line.

4. Find the distance between the point $(1, 1, 1)$ and the plane $x + 2y + 3z = 1$. Also find the point on the plane that is the closest to $(1, 1, 1)$.

5. (Harder, if we have time.) Consider three planes:

$$\begin{aligned} P_1 &: 2x + y = 0 \\ P_2 &: x - y + 5z = 0 \\ P_3 &: -x - y + 2z = 0. \end{aligned}$$

   Find their intersection(s). (That is, if it's a point, find that point; if it's a line, find the equation to that line; if it's a plane, find the equation to that plane.)

   Ans: the only intersection point is $(0, 0, 0)$. But, why? Why aren't there any other?