18.02A Recitation — Monday, Dec. 10, 2018

Quick Review

- Surface area. Let $z = f(x, y)$ represent a graph. Let R be a region in the $x - y$ plane. The surface area of the graph $z = f(x, y)$ over the region R is given by

$$\iint_R \sqrt{1 + (f_x)^2 + (f_y)^2} dA.$$

- Suppose $u = u(x, y)$ and $v = v(x, y)$.

$$\frac{\partial(u, v)}{\partial(x, y)} = \left|\det \begin{pmatrix} u_x, u_y \\ v_x, v_y \end{pmatrix}\right|,$$

$$\frac{\partial(x, y)}{\partial(u, v)} = \left|\det \begin{pmatrix} x_u, x_v \\ y_u, y_v \end{pmatrix}\right|.$$

  The above two quantities are reciprocal to each other; that is:

$$\frac{\partial(u, v)}{\partial(x, y)} \cdot \frac{\partial(x, y)}{\partial(u, v)} = 1.$$

  You can use this relation to find both quantities by only finding the easier of the two.

- Change of variable. Suppose initially you have a double integral

$$\iint_R f(x, y) dxdy.$$

  The change of variable is

$$u = u(x, y), v = v(x, y), \tag{1}$$

  and you need to rewrite the double integral in u, v coordinates.

  1. Figure out the region R' in the $u - v$ plane corresponding to R in the $x - y$ plane.
  2. Compute $\frac{\partial(x,y)}{\partial(u,v)}$. (If it's hard to solve x, y in terms of u, v according to (1), try computing $\frac{\partial(u,v)}{\partial(x,y)}$ and then take the reciprocal.)
  3. The integral now becomes

$$\iint_{R'} f(x, y) \frac{\partial(x, y)}{\partial(u, v)} dudv.$$

  4. Convert any remaining x, y in the integral to u, v based on (1). Now you should have a double integral **completely** in u, v. (You should not see any x, y in your integral!!)
  5. Carry out the double integral in u, v like we practiced before.

Practice problems:

1. Evaluate
$$\iint_R \left(\frac{x-y}{x+y+2}\right)^2 dxdy$$
, where R is the parallelogram with $(1,0)$, $(-1,0)$, $(0,1)$ and $(0,-1)$ as its four vertices. Use the change of variable
$$u = x+y, v = x-y.$$

2. Evaluate
$$\iint_R (2x-3y)^2(x+y)^2 dxdy,$$
where R is the triangle bounded by the positive x-axis, the negative y-axis, and the line $2x-3y=4$, by making the change of variable $u = x+y$, $v = 2x-3y$.

3. Find the surface area of the plane $z = ax+by$ over an arbitrary region R with area(R) $=$ c.

4. Find the surface area of the parabloid $z = 1 - ax^2 - ay^2$ where $0 < a < 1$ over the unit disk $x^2 + y^2 \leq 1$.