18.02A Recitation — Monday, Oct. 21, 2018

Quick Review

- Vectors: direction + magnitude
- Operations:
  1. Vector addition
     Geometrically, *diagonal of the parallelogram formed by the two vectors.*
     Algebraically, *addition by component* $\langle x_1, y_1, z_1\rangle + \langle x_2, y_2, z_2\rangle = \langle x_1 + x_2, y_1 + y_2, z_1 + z_2\rangle$.
  2. Scalar multiplication
     Geometrically, *elongate/shorten* a vector.
     Analytically, *scalar multiplication by component* $\lambda \cdot \langle x, y, z\rangle = \langle \lambda x, \lambda y, \lambda z\rangle$.
- Length of a vector $||\langle x, y, z\rangle|| = \sqrt{x^2 + y^2 + z^2}$.
- Dot product

  Geometrically, $\mathbf{v} \cdot \mathbf{w} = ||\mathbf{v}||||\mathbf{w}|| \cos\theta$, where θ is the angle formed by $\mathbf{v}$ and $\mathbf{w}$.

  Algebraically,
  $$\langle x_1, y_1, z_1\rangle \cdot \langle x_2, y_2, z_2\rangle = x_1x_2 + y_1y_2 + z_1z_2.$$
- Dot product and length $||\mathbf{v}|| = \sqrt{\mathbf{v} \cdot \mathbf{v}}$.

Practice problems:

1. Find the point on the y-axis that is equidistance from $\langle 2, 5, -3\rangle$ and $\langle -3, 6, 1\rangle$. (*equidistance* means same distance)

2. If $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, use vectors to find the coordinates of the mid point of P_1 and P_2.

   What about the same question for two points $P_1 = (x_1, y_1, z_1)$ and $P_2 = (x_2, y_2, z_2)$ in 3D?

3. Use vectors to show that the two diagonals of a parallelogram disect each other.